

SUBJECTIVE SOLVED EXAMPLES

Example - 1 A copper and a tungsten (W) plate having a thickness $\delta = 2 \text{ mm}$ each are riveted together so that at 0°C they form a flat bimetallic plate. Find the average radius of the curvature of this plate at $t = 200^\circ\text{C}$. The coefficient of linear expansion for copper and tungsten are $\alpha_{\text{Cu}} = 1.7 \times 10^{-5}/\text{K}$ and $\alpha_{\text{W}} = 0.4 \times 10^{-5}/\text{K}$.

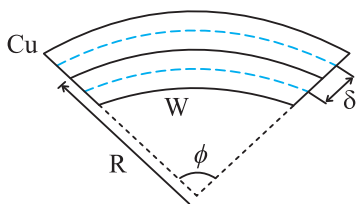
SOLUTION :

The average length of copper plate at a temperature

$$T = 200^\circ\text{C} \text{ is } \ell_c = \ell_0(1 + \alpha_c T),$$

where ℓ_0 is the length of copper plate at 0°C .

The length of the tungsten plate is $\ell_t = \ell_0(1 + \alpha_t T)$.



We shall assume that the edges of plates are not displaced during deforming and that an increase in the plate thickness due to heating can be neglected.

From figure we have :

$$\ell_c = \phi(R + \delta/2)$$

$$\ell_t = \phi(R - \delta/2)$$

Consequently,

$$\phi(R + \delta/2) = \ell_0(1 + \alpha_c T) \quad \dots\dots(i)$$

$$\phi(R - \delta/2) = \ell_0(1 + \alpha_t T) \quad \dots\dots(ii)$$

To eliminate the unknown quantities, ϕ and ℓ_0 we divide the equation (i) by (ii) term wise :

$$\Rightarrow \frac{(R + \delta/2)}{(R - \delta/2)} = \frac{(1 + \alpha_c T)}{(1 + \alpha_t T)}$$

$$\Rightarrow R = \delta \frac{[2 + (\alpha_c + \alpha_t) T]}{[2(\alpha_c - \alpha_t) T]}$$

$$\Rightarrow R = \frac{\delta}{(\alpha_c - \alpha_t) T}$$

Neglecting $(\alpha_c + \alpha_t)$ in the numerator as $(\alpha_c + \alpha_t) < 1$

Substituting the values in above relation we get :

$$R = 0.769 \text{ m.}$$

Example - 2 A sinker of weight W_0 has an apparent weight W_1 when placed in a liquid at a temperature T_1 and W_2 when weighed in the same liquid at a temperature T_2 . The coefficient of cubical expansion of the material of the sinker is β . What is the coefficient of volume expansion of the liquid ?

SOLUTION :

Let $\theta = T_2 - T_1$ and γ = coefficient of volume expansion of liquid.

Let density of liquid at temperatures T_1 and T_2 be ρ_1 and ρ_2 respectively.

$$\Rightarrow \rho_1 = \rho_2(1 + \gamma\theta) \quad \dots\dots(i)$$

Let V_1 and V_2 be the volumes of the sinker at temperatures T_1 and T_2 respectively.

$$\Rightarrow V_2 = V_1(1 + \beta\theta) \quad \dots\dots(ii)$$

The loss in weight at $T_1 = V_1 \rho_1 g$

$$\Rightarrow W_0 - W_1 = V_1 \rho_1 g \quad \dots\dots(iii)$$

The loss in weight at $T_2 = V_2 \rho_2 g$

$$\Rightarrow W_0 - W_2 = V_2 \rho_2 g \quad \dots\dots(iv)$$

Dividing (iii) and (iv) we get :

$$\frac{W_0 - W_1}{W_0 - W_2} = \frac{V_1 \rho_1}{V_2 \rho_2}$$

Using (i) and (ii) we get :

$$\frac{W_0 - W_1}{W_0 - W_2} = \frac{1 + \gamma \theta}{1 + \beta \theta} \Rightarrow \gamma = \left(\frac{W_2 - W_1}{W_0 - W_2} \right) \frac{1}{T_2 - T_1} + \left(\frac{W_0 - W_1}{W_0 - W_2} \right) \beta$$

$$\Rightarrow 1 + \gamma \theta = \frac{W_0 - W_1}{W_0 - W_2} + \beta \left(\frac{W_0 - W_1}{W_0 - W_2} \right) \theta$$

Example - 3 A vessel is filled completely with 500 gm of water on top of 1000 gm of mercury. When 21200 cal of heat is given to it, water of mass 3.52 gm overflows. Calculate the coefficient of volume expansion of mercury. The expansion of the vessel may be neglected. Coefficient of volume expansion of water = $1.5 \times 10^{-4}/^\circ\text{C}$, density of mercury = 13.6 g/cc, density of water = 1 g/cc and specific heat of mercury = 0.03 cal/g/ $^\circ\text{C}$.

SOLUTION :

Let γ_ℓ = coeff. of volume expansion of Hg and
 γ_w = coeff. of volume expansion of water and
 θ = rise in temperature.

Water overflow = (final volume of water)
 + (final volume of Hg) – (final volume of vessel)

Initial volumes of water and mercury are 500 ml
 and 1000/13.6 ml respectively.

$\Rightarrow$ water overflow

$$= \frac{500}{1} (1 + \gamma_w \theta) + \frac{1000}{13.6} (1 + \gamma_\ell \theta) - \left(\frac{500}{1} + \frac{1000}{13.6} \right)$$

Note that the volume of vessel remains same as its expansion is neglected.

Now water overflow

$$= \frac{3.52 \text{ gm}}{d_{\text{final}}} = \frac{3.52}{1} (1 + \gamma_w \theta)$$

$$= 500 (1 + \gamma_w \theta)$$

$$+ \frac{1000}{13.6} (1 + \gamma_\ell \theta) - \left(500 + \frac{1000}{13.6} \right) \dots (i)$$

Another equation can be formed by calorimetry :

Heat gained by water and Hg = 21200 cal.

$$\Rightarrow 500 \times 1 \times \theta + 1000 \times 0.03 \times \theta = 21200$$

$$\Rightarrow \theta = 40^\circ\text{C}$$

Substituting the value of θ in equation (i), we get :

$$3.52 (1 + 40 \gamma_w) = 500 \times 40 \gamma_w + \frac{1000}{13.6} \gamma_\ell (40)$$

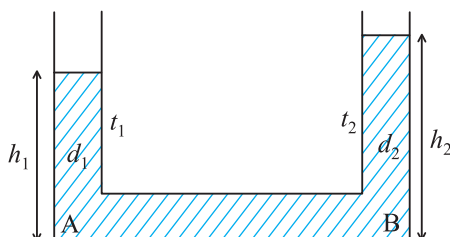
$$\Rightarrow \gamma_\ell = \frac{(3.52 \times 1.006 - 3) \times 13.6}{40000}$$

$$\Rightarrow \gamma_\ell = 1.84 \times 10^{-4} (^\circ\text{C})^{-1}$$

Example - 4 A U tube contains mercury. The left limb of the tube is maintained at a temperature of $T_1^\circ\text{C}$ and the right limb at a temperature of $T_2^\circ\text{C}$. The heights of mercury columns in the left and right limbs are h_1 and h_2 respectively. Find the coefficient of volume expansion of Hg. Neglect the expansion of tube.

SOLUTION :

Let d_1 and d_2 be the densities of mercury in the left and right limbs respectively.



By hydrostatic conditions :

$$\begin{aligned}
 P_A &= P_B & \Rightarrow \frac{h_1}{h_2} &= \frac{d_2}{d_2(1 + \gamma_\ell \theta)} \\
 P_{atm} + h_1 d_1 g &= P_{atm} + h_2 d_2 g & \Rightarrow 1 + \gamma_\ell \theta &= \frac{h_2}{h_1} \\
 \Rightarrow \frac{h_1}{h_2} &= \frac{d_2}{d_1} & \Rightarrow \gamma_\ell &= \frac{h_2 - h_1}{h_1 \theta}
 \end{aligned}$$

Example - 5 The rods of copper, brass and steel are welded together to form a Y-shaped structure. The cross-sectional area of each rod is 4 cm^2 . The end of copper rod is maintained at 100°C and the ends of the brass and steel rods at 0°C . Assume that there is no loss of heat from the surfaces of the rods. The lengths of rods are: copper 46 cm, brass 13 cm and steel 12 cm.

(a) What is the temperature of the junction point ?

(b) What is the heat current in the copper rod ?

$K(\text{Cu}) = 0.92$, $K(\text{steel}) = 0.12$ and $K(\text{brass}) = 0.26 \text{ C.G.S. units}$.

SOLUTION :

Let θ be the temperature of the junction.

Let H = Heat flow per second in copper rod.

H_1 = Heat flow per second in steel rod.

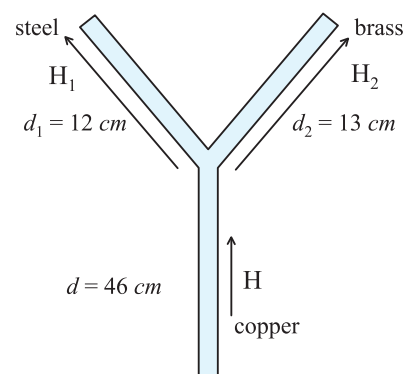
H_2 = Heat flow per second in brass rod.

$$\Rightarrow H = H_1 + H_2$$

$$\frac{K_{\text{Cu}} A (100 - \theta)}{d} = \frac{K_1 A (\theta - 0)}{d_1} + \frac{K_2 A (\theta - 0)}{d_2}$$

$$\frac{0.92 \times 4 \times (100 - \theta)}{46} = \frac{0.12 \times 4 \times \theta}{12} + \frac{0.26 \times 4 \times \theta}{13}$$

$$\Rightarrow \theta = 40^\circ\text{C}$$



$$\begin{aligned}
 H_{\text{cu}} &= \frac{K_{\text{Cu}} A (100 - \theta)}{d} = \frac{0.92 \times 4 \times (100 - 40)}{46} \\
 &= 4.8 \text{ cal/s.}
 \end{aligned}$$

Example - 6 A steam pipe 1 m in length with an outside diameter of 5.00 cm has a uniform temperature of 100°C . The pipe is insulated with a 6.00 cm layer of asbestos fibre. If the room temperature is 20°C , what is the heat loss to room per hour per meter of pipe length ? What is the temperature in the middle of the layer of insulation ? $K(\text{asbestos}) = 0.19 \times 10^{-4} \text{ cal/s/}^\circ\text{C/m}$.

SOLUTION :

The system has cylindric symmetry, so we conclude that the isothermal surfaces are concentric cylinders and that heat flows radially outward along lines parallel to the normal vectors.

Using the expression for rate of heat transfer per second :

$$\frac{dQ}{dt} = \frac{2\pi L K (\theta_1 - \theta_2)}{\log \frac{r_2}{r_1}}$$

$$\Rightarrow \frac{dQ}{dt} = \frac{2\pi \times 1 \times 0.19 \times 10^{-4} \times (100 - 20)}{\log \frac{8.5}{2.5}}$$

$$\Rightarrow \frac{dQ}{dt} = 7.80 \times 10^{-3} \text{ cal/s} = 28.1 \text{ cal/hr.}$$

To find the temperature in the middle of layer of insulation, we will use the following relation. Let θ_r be the temperature of the isothermal surface at a radial distance r .

$$\theta_r = \theta_1 - (\theta_1 - \theta_2) \frac{\log \left(\frac{r}{r_1} \right)}{\log \left(\frac{r_2}{r_1} \right)}$$

$$\Rightarrow \theta_r = 100 - (100 - 20) \frac{\log \left(\frac{5.5}{2.5} \right)}{\log \left(\frac{8.5}{2.5} \right)}$$

$$\Rightarrow \theta_r = 48.457^\circ\text{C}$$

To derive the expression for θ_r , equate the rate of heat flow in the region $r = r_1$ to $r = r_2$ with the rate in the region $r = r_1$ to the radial distance r .

$$\frac{2\pi KL(\theta_1 - \theta_2)}{\log r_2 / r_1} = \frac{2\pi KL(\theta_1 - \theta_r)}{\log r / r_1}$$

Example - 7

The rate at which the radiant energy reaches the surface of earth from the sun is about 1.4 kW/m^2 .

The distance from earth to the sun is about $1.5 \times 10^{11} \text{ m}$, and the radius of sun is about $0.7 \times 10^9 \text{ m}$.

(a) What is the rate of radiation of energy, per unit area, from the sun's surface?

(b) If the sun radiates as an ideal blackbody, what is the temperature of its surface?

SOLUTION :

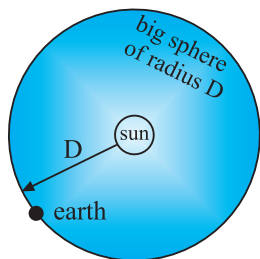
Let D = distance from SUN to the earth = $1.5 \times 10^{11} \text{ m}$.

Let R = radius of SUN = $0.7 \times 10^9 \text{ m}$.

Let power of SUN = P = energy radiated from the surface of SUN per second.

- (a) Hence in every one second, P joules of energy are radiated from the surface of SUN and this energy passes through a big sphere of radius D centred at SUN.

Hence at the circumference of this big sphere (i.e. near the surface of the earth), the energy crossing through a unit area per sec.



$$= \frac{P}{\text{area of big sphere}} = \frac{P}{4\pi D^2}$$

$$\frac{P}{4\pi D^2} = 1.4 \times 10^3 \text{ W m}^{-2}$$

$$\Rightarrow P = 4\pi (1.5 \times 10^{11})^2 \times 1.4 \times 10^3 \text{ W}$$

$$P = 3.96 \times 10^{26} \text{ W}$$

Rate of radiation of energy per sec. per unit area of SUN's surface is given by :

$$\frac{P}{\text{area of sun}} = \frac{P}{4\pi R^2} = 6.43 \times 10^7 \text{ W m}^{-2}$$

- (b) If SUN is an ideal black body, $\epsilon = 1$.

$$\Rightarrow E = \sigma T^4$$

$$\Rightarrow T = \left(\frac{E}{\sigma} \right)^{1/4} = \left(\frac{6.43 \times 10^7}{5.67 \times 10^{-8}} \right)^{1/4} = 5803 \text{ K}$$

Example - 8

An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C . The specific heat capacity c of the container varies with temperature T according to the empirical relation $c = A + BT$, where $A = 100 \text{ cal/kg-K}$ and $B = 2 \times 10^{-2} \text{ cal/kg-K}^2$. If the final temperature of the container is 27°C , determine the mass of the container. (Latent heat of fusion for water = $8 \times 10^4 \text{ cal/kg}$, specific heat capacity of water = 10^3 cal/kg-K).

SOLUTION :

Heat gained by ice in converting into liquid water at 0°C is

$$q_1 = mL = (0.1 \text{ kg}) (8 \times 10^4 \text{ cal/kg}) = 8 \times 10^3 \text{ cal}$$

Heat gained by liquid water in raising its temperature from 0°C to 27°C is

$$\begin{aligned} q_2 &= mc \Delta T = (0.1 \text{ kg}) (10^3 \text{ cal kg}^{-1} \text{ K}^{-1}) (27 \text{ K}) \\ &= 2.7 \times 10^3 \text{ cal} \end{aligned}$$

Total heat gained by water is

$$\begin{aligned} q &= q_1 + q_2 = 8 \times 10^3 \text{ cal} + 2.7 \times 10^3 \text{ cal} \\ &= 10.7 \times 10^3 \text{ cal} \quad \dots\dots(i) \end{aligned}$$

Heat lost by the container in cooling from $T_1 (= 500 \text{ K})$ to $T_2 (= 300 \text{ K})$ is

$$\begin{aligned} q' &= \int_{T_1}^{T_2} mc \, dT = m \int_{T_1}^{T_2} (A + BT) \, dT \\ &= m \left[A (T_2 - T_1) + (B/2) (T_2^2 - T_1^2) \right] \end{aligned}$$

Substituting the given data, we get :

$$\begin{aligned} q' &= m [(10^2 \text{ cal kg}^{-1} \text{ K}^{-1}) (500 \text{ K} - 300 \text{ K}) \\ &\quad + (2 \times 10^{-2} \text{ cal kg}^{-1} \text{ K}^{-2}/2) \{ (500 \text{ K})^2 - (300 \text{ K})^2 \}] \\ &= m [2 \times 10^4 \text{ cal kg}^{-1} + 1.6 \times 10^3 \text{ cal kg}^{-1}] \\ &= m (2.16 \times 10^4 \text{ cal kg}^{-1}) \quad \dots\dots(ii) \end{aligned}$$

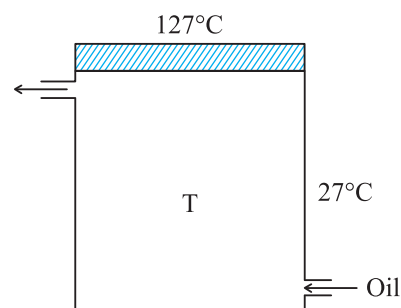
From equations (i) and (ii), we can write

$$\begin{aligned} m (2.16 \times 10^4 \text{ cal kg}^{-1}) &= 10.7 \times 10^3 \text{ cal} \\ \Rightarrow m &= \frac{(10.7 \times 10^3 \text{ cal})}{(2.16 \times 10^4 \text{ cal kg}^{-1})} = 0.495 \text{ kg} \end{aligned}$$

Example - 9

Oil at temperature T is continuously circulating through an insulating container as shown in the figure. A lid of thickness 6 mm, conductivity $0.16 \text{ J/(s m } ^\circ\text{C)}$ and emissivity 0.6 closes the top of the container. The temperature at the top of the lid is steady at 127°C while the surroundings have a temperature of 27°C . Calculate (a) the rate of heat lost per unit area due to the radiation from the lid, and (b) the temperature of the oil.

Given : Stefan-Boltzmann constant, is $\sigma = (17/3) \times 10^{-8} \text{ W/m}^2 \text{ K}^4$.

**SOLUTION :**

The rate of heat lost per unit area of the lid

$$\begin{aligned} &= \frac{1}{A} \frac{dQ}{dt} = e\sigma (T_2^4 - T_1^4) \\ &= (0.6) \times 17/3 \times 10^{-8} \times \{ (273 + 127)^4 - (273 + 27)^4 \} \\ &= (0.6) (17/3) (10^{-8}) (2.56 \times 10^{10} - 8.1 \times 10^9) \text{ W/m}^2 \\ &= 595 \text{ W/m}^2 \end{aligned}$$

The above heat lost is supplied by the circulating oil through the lid whose one side temperature is 127°C and the other side is T .

The rate of heat flow through the lid

$$\begin{aligned} &= \frac{1}{A} \frac{dQ}{dt} = k \frac{\Delta T}{l} = (0.16) \frac{(T - 127)}{(6 \times 10^{-3})} \\ &= 25 (T - 127) \end{aligned}$$

Equating this to 595 W/m^2 , we get :

$$T = \left(\frac{595}{25} + 127 \right) ^\circ\text{C} = 150.8^\circ\text{C}$$

Example - 10 A wire of length ℓ and radius r has a weight W and the Young's modulus Y . It is suspended vertically from a fixed point. Calculate the increase in length of wire produced due to its own weight W .

SOLUTION :

As tension is non-uniform along the wire, we have variable stress. Consider an infinitesimal element of length dx at a distance x from bottom end A.

The tension at this position in the wire = $\frac{mgx}{\ell}$

Let $d\ell$ = increase in length of the element.

$$\Rightarrow \text{strain} = \frac{d\ell}{dx}$$

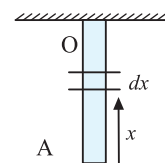
$$\Rightarrow Y = \frac{mgx/\ell A}{d\ell/dx} \Rightarrow d\ell = \frac{mgx dx}{Y\ell A}$$

The total change in length of the wire is the sum of the changes in length of infinite such elements along the wire.

$$\text{Total change in length} = \Delta\ell = \int_0^\ell d\ell = \int_0^\ell \frac{mgx dx}{Y\ell A}$$

$$\Delta\ell = \frac{mg}{Y\ell A} \frac{\ell^2}{2} \Rightarrow \Delta\ell = \frac{mgl}{2Y\pi r^2}$$

$$\Rightarrow \Delta\ell = \frac{(\text{stress at mid pt.})}{Y}$$

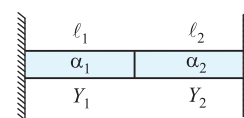


Example - 11 Two rods of different metals having the same area of cross-section A , and same length ' ℓ ' are placed end to end between two massive walls as shown. The values of Young's modulus, length and coefficient of linear expansion for the rods are given.

The temperature of both the rods is now raised by T degrees.

(a) Find the force with which the rods act on each other.

(b) Find the length of the rods at higher temperature.



SOLUTION :

Let $\Delta\ell_1$ = change in length due to thermal expansion in rod 1.

ΔL_1 = change in length due to elastic forces in rod 1.

Let $\Delta\ell_2$ and ΔL_2 be the respective values for rod 2.

(Note that a decrease in length will mean negative change in length). As the walls are fixed,

Net change in length of rod 1 + Net change in length of rod 2 = 0

$$\Rightarrow (\Delta\ell_1 + \Delta L_1) + (\Delta\ell_2 + \Delta L_2) = 0$$

$$\left(\ell_1 \alpha_1 T + \frac{F\ell_1}{AY_1} \right) + \left(\ell_2 \alpha_2 T + \frac{F\ell_2}{AY_2} \right) = 0$$

where F is the force of compression between rods.

$$\Rightarrow F = \frac{-Y_1 Y_2 AT (\ell_1 \alpha_1 + \ell_2 \alpha_2)}{Y_1 \ell_2 + Y_2 \ell_1}$$

Final length of rod 1 = $\ell_1 + \Delta\ell_1 + \Delta L_1$

$$= \ell_1 + \ell_1 \alpha_1 T + \frac{F\ell_1}{AY_1}$$

$$= \ell_1 + \frac{(Y_1 \alpha_1 - Y_2 \alpha_2) T}{\left(\frac{Y_1}{\ell_1} + \frac{Y_2}{\ell_2} \right)}$$

Final length of rod 2 = $\ell_2 + \Delta\ell_2 + \Delta L_2$

$$= \ell_2 + \ell_2 \alpha_2 T + \frac{F\ell_2}{AY_2}$$

$$= \ell_2 - \frac{(Y_1 \alpha_1 - Y_2 \alpha_2) T}{\left(\frac{Y_1}{\ell_1} + \frac{Y_2}{\ell_2} \right)}$$

Note that the displacement of the junction of rods is :

$$\frac{(Y_1 \alpha_1 - Y_2 \alpha_2) T}{\left(\frac{Y_1}{\ell_1} + \frac{Y_2}{\ell_2} \right)} \quad (\text{towards right if +ve}).$$